INTERNATIONAL JOURNAL FOR NUMERICAL AND ANALYTICAL METHODS IN GEOMECHANICS, VOL. 21, 477–484 (1997)

SHORT COMMUNICATIONS

UPPER BOUND SOLUTION FOR PULLOUT CAPACITY OF ANCHORS ON SANDY SLOPES

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SUMMARY

By making use of limit analysis, an upper bound solution in a closed form for determining the ultimate pullout capacity of plate anchors buried in sandy slopes has been established. The anchor plate orientation has been considered either horizontal or parallel to the slope, with the pullout force applied perpendicular to the plate. It has been found that the pullout capacity for horizontal anchors, even on slopes, remains the same as that on horizontal ground surface as long as the average embedment ratio is kept constant. Whereas for anchors which are aligned parallel to the slope the collapse load decreases continuously with the increase in the inclination of slope. © 1997 by John Wiley & Sons, Ltd.

Int. J. Numer. Anal. Meth. Geomech., Vol. 21, 477–484 (1997)
(No. of Figures: 4 No. of Tables: 0 No. of Refs: 13)

Key words: anchors; failure surface; foundations; limit analysis; sand; slopes

INTRODUCTION

Anchors are employed often to resist uplift tensile forces occurring in the foundations of structures such as transmission towers, drydocks, pipelines under water, etc. Considerable research has been performed by various investigators to determine both the load deformation response of anchors under working load conditions[1–3] (mainly on the basis of the theory of elasticity) and the pullout capacity of anchors in the ultimate state of failure. Among the approaches used in the later case are the limit equilibrium method,[4] the theory of expansion of cavities,[5] the finite element method,[6] limit analysis[7] and the method of characteristics.[8] A comparison of these different theoretical methods is given by Kumar.[9] A number of experimental model tests have also been conducted to examine the load–deformation relationships of anchors[10,11] using conventional 1–*g* as well as centrifugal testing methods. However, the available studies, both theoretical and experimental, are limited to anchors embedded below a horizontal ground surface. On the other hand, hardly any effort has been so far to investigate the performance of anchors placed on slopes. The present contribution provides an approximate

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CCC 0363–9061/97/070477–08$17.50

Received 22 January 1996
Revised 30 September 1996

solution using the upper bound method of limit analysis, for determining the ultimate pullout capacity of strip anchors embedded below an inclined ground surface. The soil surrounding the anchor has been considered as cohesionless and is assumed to obey an associated flow rule. The anchor plate has been oriented either horizontally or parallel to the slope, wherein the pullout force has been applied in a direction normal to the plate. A comparison of the results obtained has been made with the available solutions for anchors placed below a horizontal ground surface.

METHOD

If the soil is assumed to follow an associated flow rule, the theorems of limit analysis then become applicable. Accordingly, the solution for any stability problem can be bracketed using the upper and lower bound methods of limit analysis. The assumption of an associated flow rule generally invokes more dilatancy than is observed in real soils. However, the upper bound solution which implicitly includes this approximation is usually only a little higher than corresponding solutions for non-associated flow rule materials.[12]

While obtaining an upper bound solution for the vertical uplift capacity of anchors placed horizontally below a horizontal ground surface, Murray and Geddes[7] demonstrated that simple assumptions of linear rupture surfaces result in reasonably acceptable solutions. Their results were found to compare well with the widely used solution of Meyerhof and Adams,[4] who used passive earth pressure coefficients for curved failure surfaces.[13]

It is being assumed herein that linear rupture surfaces remain applicable for determining the upper bound solution for anchors placed on slopes. Further, in order to employ the theorems of limit analysis, the soil material has been assumed to obey an associated flow rule. The collapse load for any trial kinematically admissible failure mechanism is obtained by equating the rate of work done by external forces, including the effect of body forces, to that of the rate of dissipation of internal energy, which in the case of perfectly cohesionless material is zero. The critical failure mechanism for minimum breakout load is searched for various possible kinematically admissible failure mechanisms.

ANALYSIS

1. Case I. Anchors embedded horizontally

Consider an anchor of width b placed horizontally at a vertical distance d from the ground, which has an inclination w with the horizontal, as shown in Figure 1(a). The anchor is pulled out vertically upward, and at failure, a failure mechanism of three triangular wedges bounded by linear rupture surfaces has been considered. Any selected failure mechanism for a given embedment ratio, $\lambda = d/b$, of the anchor can be fully defined by means of just three independent variables namely α_1, α_2 and x; wherein α_1 and α_2 are the inclinations of the failure surfaces AC and BD with the horizontal and x is the distance OP as indicated in Figure 1. At the ultimate failure state, the wedge OAB, having weight W_0, is assumed to move along with the plate in the direction of pullout, which is vertical, at a virtual velocity V_0. The two adjoining wedges OAC and OBD, with weights W_1 and W_2, move with velocities V_1 and V_2 but making an angle ϕ, the friction angle of soil, with the linear failure surfaces AC and BD. V_{01} and V_{02} are the relative velocities of the wedges OAC and OBD with respect to that of the wedge OAB. The relative

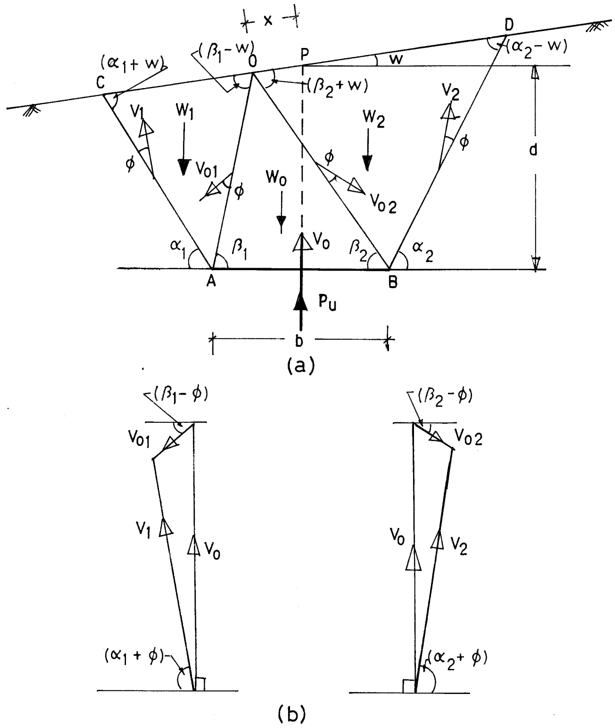

Figure 1. Failure mechanism and velocity diagram for horizontal anchors

velocities are inclined at an angle ϕ with the linear interfaces OA and OB. The relationships, as given below, among these various velocities can be established using Figure 1(b).

$$V_1 = V_0 \cos(\beta_1 - \phi)/\sin(\alpha_1 + \beta_1) \tag{1a}$$

$$V_{01} = V_0 \cos(\alpha_1 + \phi)/\sin(\alpha_1 + \beta_1) \tag{1b}$$

$$V_2 = V_0 \cos(\beta_2 - \phi)/\sin(\alpha_2 + \beta_2) \tag{1c}$$

$$V_{02} = V_0 \cos(\alpha_2 + \phi)/\sin(\alpha_2 + \beta_2) \tag{1d}$$

wherein β_1 and β_2 are the inclinations of interfaces OA and OB with the horizontal.

In order that a chosen failure mechanism remains kinematically admissible, the following conditions must be obeyed:

(i) $(\pi/2 - \phi) > [\alpha_1 \text{ and } \alpha_2] > w$; and
(ii) $(\pi/2 + \phi) > [\text{angles } \beta_1 \text{ and } \beta_2] > w$.

Also, as the soil is considered as cohesionless, the inclination of slope $w < \phi$ so that the slope itself remains always stable.

Equating the rate of work done by external forces to the rate of dissipation of internal energy, which is zero for the present case, and on the simplification, it can be shown that the pullout force

P_u per unit length of anchor plate will become

$$P_u = W_0 + W_1 \cdot (V_1/V_0) \sin(\alpha_1 + \phi) + W_2 \cdot (V_2/V_0) \sin(\alpha_2 + \phi) \tag{2}$$

On further expansion, the magnitude of P_u can be expressed in the following form:

$$P_u = \gamma b^2/2 \cdot [f_0(\beta_1, \beta_2) + f_1(\beta_1, \beta_2) \cdot g_1(\alpha_1) + f_2(\beta_1, \beta_2) \cdot g_2(\alpha_2)] \tag{3}$$

in which,

$$\gamma = \text{soil density}$$

$$f_0(\beta_1, \beta_2) = \sin \beta_1 \sin \beta_2 / \sin(\beta_1 + \beta_2)$$

$$f_1(\beta_1, \beta_2) = \sin^2 \beta_2 \sin(\beta_1 - w) \cos(\beta_1 - \phi)/\sin^2(\beta_1 + \beta_2)$$

$$f_2(\beta_1, \beta_2) = \sin^2 \beta_1 \sin(\beta_2 + w) \cos(\beta_2 - \phi)/\sin^2(\beta_1 + \beta_2)$$

$$g_1(\alpha_1) = \sin(\alpha_1 + \phi)/\sin(\alpha_1 + w)$$

$$g_2(\alpha_2) = \sin(\alpha_2 + \phi)/\sin(\alpha_2 - w)$$

As the chosen failure mechanism, for a given soil type and for given embedment ratio λ, depends on the values of three independent variables α_1, α_2 and x. Therefore, amongst the various possible kinematically admissible failure mechanisms, the failure load P_u must obviously be minimized with respect to admissible variations of α_1, α_2 and x.

Carrying out partial differentation of P_u with respect to α_1 and α_2,

$$\partial P_u/\partial \alpha_1 = (\gamma b^2/2) \cdot f_1(\beta_1, \beta_2 \cdot (\partial g_1/\partial \alpha_1) \tag{4a}$$

$$\partial P_u/\partial \alpha_2 = (\gamma b^2/2) \cdot f_2(\beta_1, \beta_2) \cdot (\partial g_2/\partial \alpha_2) \tag{4b}$$

in which

$$\partial g_1/\partial \alpha_1 = \sin(w - \phi)/\sin^2(\alpha_1 + w)$$

$$\partial g_2/\partial \alpha_2 = -\sin(w + \phi)/\sin^2(\alpha_2 - \phi)$$

It can be seen that for any failure mechanism and for a stable slope ($w < \phi$), $\partial g_1/\partial \alpha_1$ will become a negative quantity. Likewise, $\partial g_2/\partial \alpha_2$ too will become a negative quantity as the value of $(w + \phi)$ is less than 180°. Also, it can further be noticed that, for any kinematically admissible failure mechanism, the sign of functions f_1 and f_2 will remain positive. As a result, $\partial P_u/\partial \alpha_1$ and $\partial P_u/\partial \alpha_2$ will always become negative quantities, and the magnitude of P_u will therefore be minimum if the values of parameters α_1 and α_2 are the maximum admissible values, i.e. equal to $(\pi/2 - \phi)$.

For values of $\alpha_1 = \alpha_2 = \pi/2 - \phi$, it must, however, be noted from the velocity relationships [equation (1)] that $V_1 = V_2 = V_0$ and $V_{01} = V_{02} = 0$. It implies that for any admissible values of x, soil wedges OAC and OBD will just move along with the wedge OAB, in the vertical upward direction, and accordingly, the magnitude of P_u from equation (3) will simply become equal to the weight of soil wedge ABCD. Accordingly, the solution obtained on the basis of minimized value of P_u with respect to admissible variations of α_1 and α_2 becomes independent of third independent variable x. Therefore, the magnitude of P_u for $\alpha_1 = \alpha_2 = (\pi/2 - \phi)$ will become absolute minimum.

The critical failure mechanism as established from the above considerations is drawn in Figure 3(a).

2. Case II. Anchors placed parallel to the slope

The failure mechanism along with the velocity triangle diagram for an anchor placed parallel to the slope and being pulled out in a direction normal to the anchor plate, is illustrated in Figure 2. The definitions of velocities V_0, V_1, V_{01}, V_2 and V_{02} and weights W_0, W_1 and W_2, remain exactly the same as were defined before in the earlier case for horizontal anchors. The following conditions in this case must, however, be obeyed so that a failure mechanism remains kinematically admissible:

(i) $(\pi/2 - \phi - w) > \alpha_1 > w$ and $(\pi/2 - \phi + w) > \alpha_2 > w$; and

(ii) $w < \beta_1 < (\pi/2 + \phi + w)$ and $w < \beta_2 < (\pi/2 + \phi - w)$.

By carrying out the analysis in the very similar fashion as was done before for the earlier case of horizontal anchors, it is likewise found that the magnitude of P_u becomes an absolute minimum

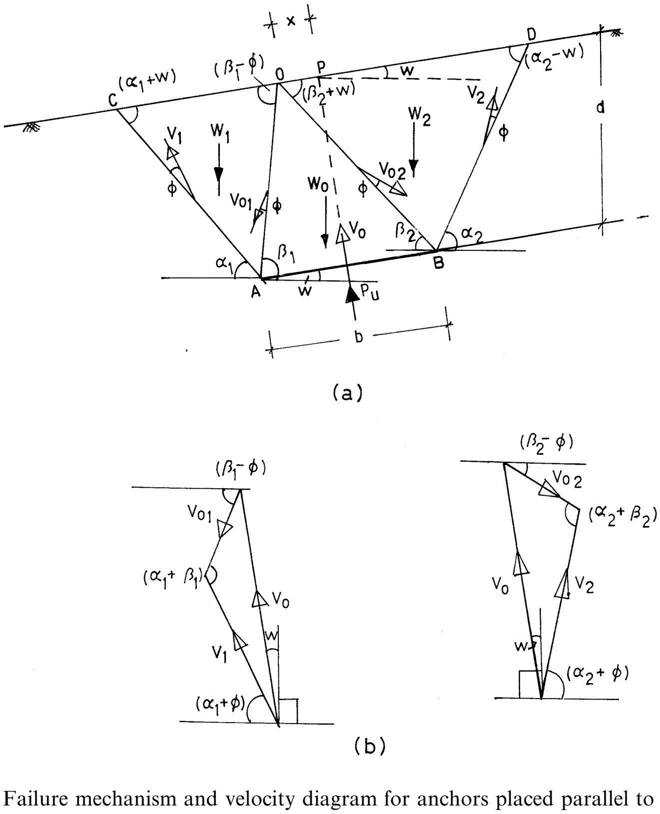

Figure 2. Failure mechanism and velocity diagram for anchors placed parallel to the slope

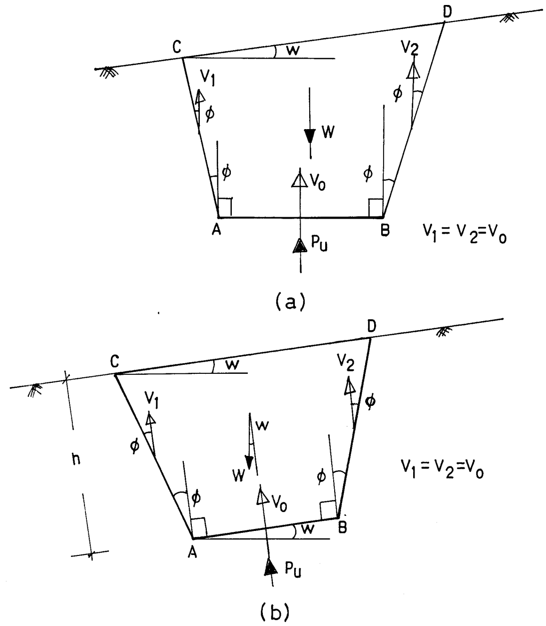

Figure 3. Critical failure mechanism for (i) horizontal anchors; and (ii) anchors placed parallel to the slope

for $\alpha_1 = (\pi/2 - \phi - w)$ and $\alpha_2 = (\pi/2 - \phi + w)$, and the variation in the third independent variable x does not affect the results.

The critical failure mechanism for this case is shown in Figure 3(b). The entire soil wedge ABCD moves just in the direction of pullout of the anchor plate and, as a result, the magnitude of the collapse load, by employing the upper bound theorem of limit analysis, becomes simply equal to the component of the weight of soil wedge ABCD in the direction of pullout.

RESULTS

The upper bound solution for determining the ultimate pullout capacity, $p_u = P_u/b$, of anchors on sandy slopes, is presented below in simple closed form.

1. *Case I. Horizontal anchors*

$$p_u = \gamma d + 0{\cdot}5\gamma b F_\gamma \tag{5}$$

In which $F_\gamma = 2\lambda^2 \tan\phi$

2. *Case II. Anchors placed parallel to the slope*

$$p_u = [\gamma h + 0{\cdot}5\gamma b F_\gamma]\, i_\gamma \tag{6}$$

Here, i_γ is the inclination factor, the value of which becomes equal to $\cos w$. The pullout factor F_γ is as defined in the case of horizontal anchors. The embedment ratio λ in this case is, however, equal to h/b; h is the normal distance ($d \cos w$) between the anchor and the ground surface as shown in Figure 3(b).

It must be noted that the direction of pullout force in both the cases is normal to the orientation of plate, and the depth d is measured in the vertical direction with reference to the mid-point of anchor.

It can be seen that the solution for horizontal anchors even on inclined ground remains the same as that on horizontal ground surface for equal values of λ. For anchors placed parallel to the slope, the ultimate pullout capacity decreases continuously with the increase in inclination of ground surface.

The variation of pullout factor F_γ with friction angle for different values of embedment ratios is illustrated in Figure 4. The magnitude of F_γ increases continuously with the increases both in the values of ϕ and λ.

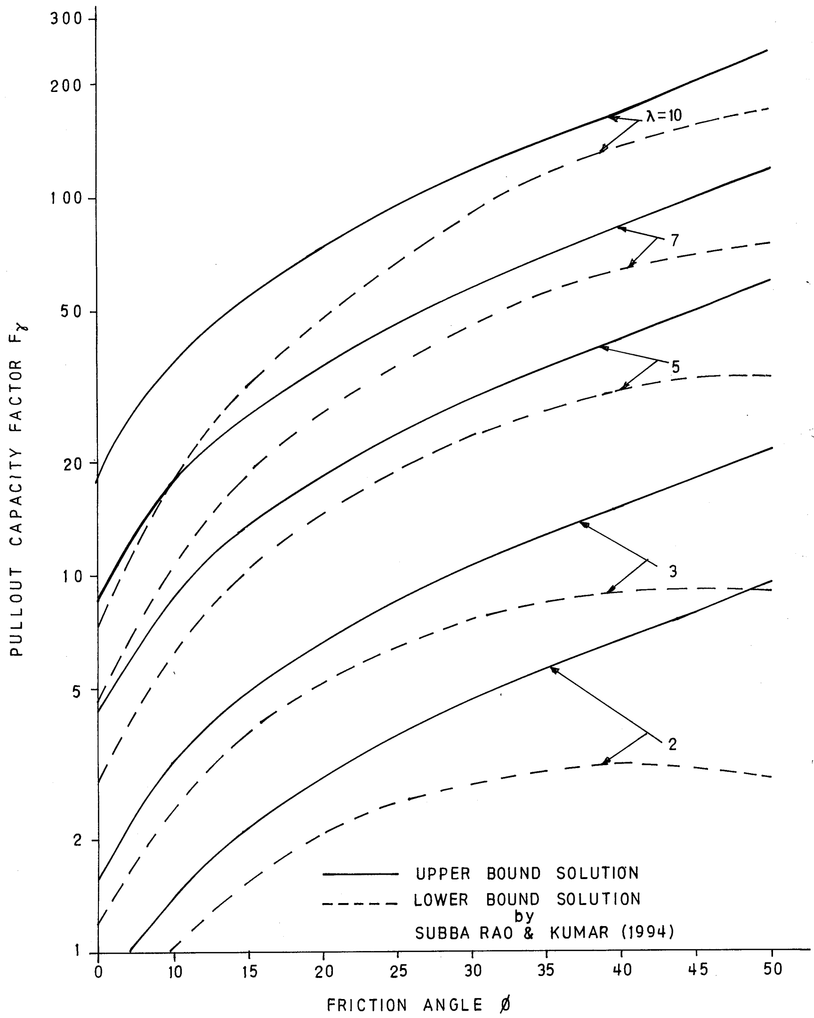

Figure 4. Variation of F_γ from (i) the upper bound solution; and (ii) the lower bound solution by using the method of charateristics for horizontal anchors placed on a horizontal ground surface

COMPARISONS

For horizontal cohesionless ground surface Murray and Geddes,[7] obtaining the vertical uplift capacity of horizontal anchors, found that the critical failure surface does make an angle ϕ with the vertical as is found from the present analysis. Their solution, although determined numerically, becomes exactly the same as given by equation (5). Also, in the case of horizontal ground surface, Meyerhof and Adams[4] provided the solution, which has been used widely, for determining the vertical uplift capacity of horizontal anchors. The comparison has shown that their solution is almost the same as given by equation (5) for horizontal ground surface. The only difference is that the value of F_γ obtained by Meyerhof and Adams is 0·95 times the corresponding value of F_γ determined from the upper bound solution for the same magnitudes of ϕ and λ.

Recent investigation using the stress characteristics method[8] has indicated that the theory of Meyerhof and Adams does give reasonably satisfactory answer in dense to very dense sand, whereas in the case of loose-to-medium dense sand the theory results overestimate the solution. For horizontal anchors placed on horizontal ground surface, the comparison of the lower bound values of factor F_γ determined by using the method of characteristics[8] with that of the corresponding upper bound solution is shown in Figure 4. The difference in the two solutions becomes more with increasing values of ϕ and lower embedment ratios.

However, the comparison for anchors exclusively on slopes could not be made as the only solutions in the literature are for horizontal ground surfaces.

CONCLUSIONS

An upper bound solution in a closed form has been presented for determining the pullout capacity of anchors buried in sandy slopes and being pulled out in a direction normal to the plate. Pullout capacity of horizontal anchors, even on slopes, remains the same as that on horizontal ground surface for equal embedment ratios λ. However, for anchors which are placed parallel to the slope, the pullout capacity decreases continuously with increase in the inclination of ground surface.

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